First International Tropospheric Airborne Measurement Evaluation Panel (TAbMEP) Meeting

Internal Estimate of Random Uncertainties

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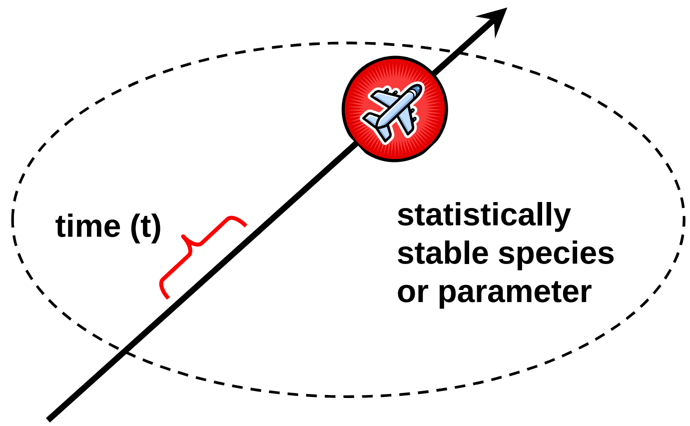
Measurement Process

- Goal: A unified database of airborne measurements (species: gas, particulate, met, and radiative) with quantified uncertainty
- During a flight, measurements of species and parameters are obtained
  - Let each measurement be x_i
  - Measurement frequency depends on instrument



- Over a subspace (temporal and spatial) of a flight we might expect to measure a statistically stable level of the species, e.g. CO
  - mean of CO (μ_{CO}) and natural variability of CO (σ_{CO})
- Within this space we can further partition into time periods of length t

What do we actually measure?

- **Goal is to estimate (μ_{CO}, σ_{CO})**

Let $\overline{x}$ be the mean of multiple measurements over time t

precision of $\overline{x}$ is a function of time (no. of samples, n)

Expectation or mean, denoted by E[ ]

$E[\overline{x}] = E[x_i] = \mu_{CO} + \delta$, where δ is the instrument calibration bias

δ is determined by comparison to a calibration standard (systematic)

δ can be a function of the level of x (e.g. nonlinear)

Variance, denoted by V[ ]

$V[x_i] = \sigma_{CO}^2 + \sigma_\varepsilon^2$, where σ_ε^2 is the instrument precision (variability)

σ_ε^2 is the random variability of the instrument

σ_ε^2 can be estimated internally during the flight, under certain assumptions

$$V[\overline{x}] = \left(\sigma_{CO}^2 + \sigma_\varepsilon^2\right)/n$$

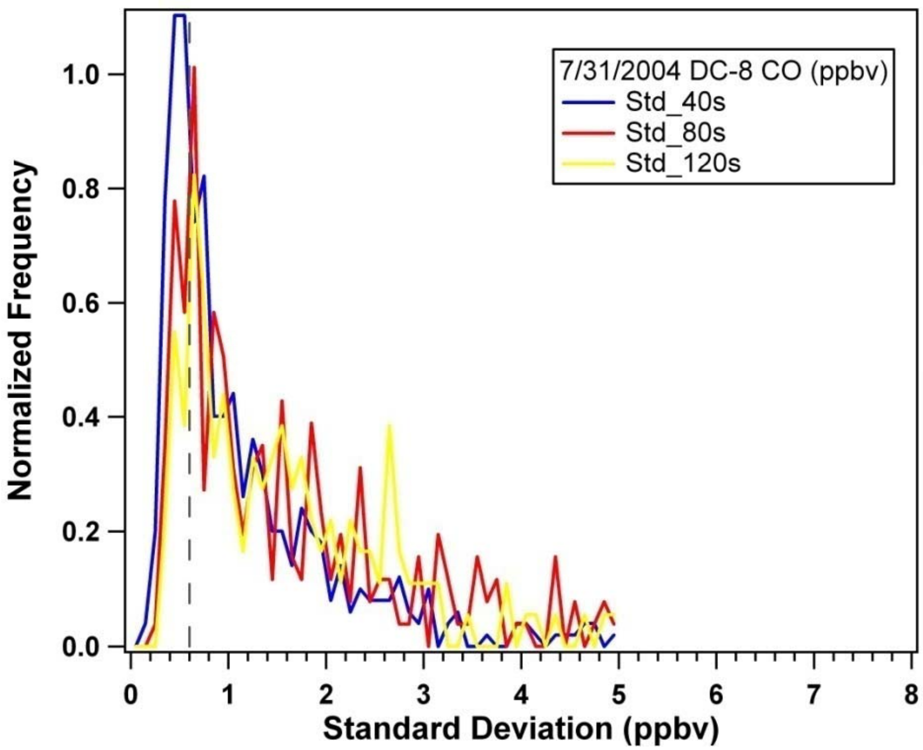
Note total measurement uncertainty, $TMU = \sqrt{\delta^2 + \sigma_\varepsilon^2}$

An Internal Estimate of Precision

- **If we choose *t* small enough, assume σ_{CO} to be small relative to σ_ε**
- **Partition flight data into subsets of size *t* and compute multiple estimates of σ_ε**
- **How long should *t* be?**
  - **depends on the temporal and spatial variability of the species or parameter of interest**
  - **depends on instrument sampling rate**
  - **requires expert judgment**
- **To quantitatively test our judgment, we can plot σ_ε estimates for varying *t*, and estimate the mean value**
  - **look for our estimate of σ_ε to be robust over small range of *t***
  - **If calibration precision is available (component of TMU), then we can compare to the internal estimate**

Internal Estimate Plot (Chen)

- Note that the mode of the distribution is relatively constant over the range of *t* from 40-120 seconds
- If the standard deviation increases with longer times, it indicates the introduction of other components of variability (due to species)
  - assumes shortest *t* was chosen to exclude species variability



How to combine multiple measurements?

- **Consider two aircrafts, let x and y be the measurements from each**

$$E[\bar{x}] = \mu_{CO} + \delta_1 \;,\; V[\bar{x}] = \sigma^2_{\varepsilon_1} \big/ n_1 \text{ (assumes } \sigma^2_{CO} = 0 \text{ over } t)$$

$$E[\bar{y}] = \mu_{CO} + \delta_2 \;,\; V[\bar{y}] = \sigma^2_{\varepsilon_2} \big/ n_2 \text{ (assumes } \sigma^2_{CO} = 0 \text{ over } t)$$

Let z be the combined (unified) estimate of CO, consider a simple average

$$z = \frac{\bar{x} + \bar{y}}{2}$$

$$E[z] = \mu_{CO} + \frac{1}{2}\delta_1 + \frac{1}{2}\delta_2$$

$$V[z] = \sigma^2_z = \frac{1}{4n_1}\sigma^2_{\varepsilon_1} + \frac{1}{4n_2}\sigma^2_{\varepsilon_2} \text{, assumes } \operatorname{cov}(x, y) = 0$$

- **Bias contribution from each instrument is reduced - still present**
- **Assumes equal weight (uncertainty) of measurements**

Weighted Combination of Measurements

- **If we have more information about the total measurement uncertainty, bias, and/or precision, consider a weighted average**

$$z = \frac{(w_1)\bar{x} + (w_2)\bar{y}}{(w_1 + w_2)} \qquad \text{if } w_1 = w_2 \text{, it becomes a simple average}$$

Let $k_1 = w_1/(w_1 + w_2)$, $k_2 = w_2/(w_1 + w_2)$

$$E[z] = \mu_{CO} + k_1\delta_1 + k_2\delta_2$$

$$V[z] = \sigma_z^2 = \frac{k_1^2}{n_1}\sigma_{\varepsilon_1}^2 + \frac{k_2^2}{n_2}\sigma_{\varepsilon_2}^2$$

assumes w's are constants, $\text{cov}(x, y) = 0$, $\sigma_{CO}^2 = 0$ over t

- **The weights could be based on calibration information or an internal estimate of precision as follows**

$$z = \frac{\left(\frac{1}{\sigma_{\varepsilon_1}^2}\right)\bar{x} + \left(\frac{1}{\sigma_{\varepsilon_2}^2}\right)\bar{y}}{\left(\frac{1}{\sigma_{\varepsilon_1}^2} + \frac{1}{\sigma_{\varepsilon_2}^2}\right)}$$

Summary

- Using knowledge of species temporal and spatial variation, allows for partitioning of the flight to isolate instrument precision
- A simple decomposition of measurements into components illustrates instrument uncertainty contributions
- Proposed a method an internal estimate of instrument precision from in-flight data, with a graphical test of validity
- Proposed a formulation of uncertainty estimates for
  - single aircraft campaigns
  - combining two (or more) instruments
- Method is generally applicable to all species and parameters
  - limitations may depend on available data